INDIAN STATISTICAL INSTITUTE
Probability Theory II: B. Math (Hons.) I
Semester II, Academic Year 2023-24
Midsem Exam

Date: 20/02/2024   **Full Marks: 50**   **Duration: 3 hours**

- **Show all your work and write explanations when needed. If you are using a result stated and/or proved in class, please quote it correctly.**

- **You are NOT allowed to use class notes, books, homework solutions, list of theorems, formulas etc.**

1. A random variable X is said to follow Laplace distribution with parameters $\mu \in \mathbb{R}$ and $\tau \in (0, \infty)$ (denoted by $X \sim \text{Laplace}(\mu, \tau)$) if X has a probability density function

$$f_X(x) = \frac{1}{2\tau} \exp\left(-\frac{|x - \mu|}{\tau}\right), \quad x \in \mathbb{R}.$$

   (a) (10 marks) Write down, with proper justification, an algorithm to simulate a random variable

   $$Z \sim \text{Laplace}(0, 1).$$

   (b) $(10 + 2 = 12 \text{ marks})$ If $Z \sim \text{Laplace}(0, 1)$, find a probability density function of $W := 1 - e^{-|Z|}$. What distributions does W follow?

2. Suppose (X, Y) is a uniformly chosen point from the region

$$\Delta := \left\{(x, y) \in \mathbb{R}^2 : \ |x| < 1 - y, \ y > 0\right\}.$$

   (a) (2 marks) Write down a joint probability density function of (X, Y).
   (b) (6 marks) Compute a marginal probability density function of X.
   (c) (6 marks) Compute a marginal probability density function of Y.
   (d) (6 marks) Calculate the cumulative distribution function of

   $$V := |X| + |Y|.$$

   (e) (6 marks) Compute a probability density function of V.
   (f) (2 marks) Are X and Y independent? Justify your answer.